LIGO probability maps and observability maps

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Introduction

We study the idea of probability maps in planning observations. The starting point are the LIGO detection maps described in Singer et al (Singer et al., 2014). The end point will be json format files containing the ra,dec, exposure time, filter, and start of exposure time. The latter are what drives the non-Obstac observations of DECam on the Blanco Telescope.

The maps are downloadable from http://www.ligo.org/scientists/first2years/

Just before the JSON file creation there is a step of summing the probability inside the all-sky DES hex layout hexes.

We start with the healpix maps of the LIGO detection probability per pixel and transform them into per pixel counterpart detection probabilites.

Limiting magnitude maps

For the case of a point source with intrinsic flux f and full width half max r faint compared to the observed sky background flux s, the signal to noise S/N is given by

$$S/N \propto \frac{T_d T_a f t}{r\sqrt{st}} = \frac{T_d T_a f \sqrt{t}}{r\sqrt{s}} \tag{1}$$

where T_d is the transmission of light through a layer of galactic dust, and T_a is the transmission of light through the Earths atmosphere. We demand a signal to noise of 10 for a detection as here a magnitude m has $\sigma_m = 0.1$, and thus a color $m_1 - m_2$ has maximum $\sigma = \sqrt(2)\sigma_m$ (assuming that the source is detected at $\geq 10\sigma$ in both filters).

We can compute a $m_{10\sigma}$ assuming fiducial conditions, then ask what is $m_{10\sigma}$ as the conditions change.

$$\frac{S}{S_f} = \frac{10\sigma}{10\sigma_f} \tag{2}$$

which in Δm is:

$$\Delta m = 2.5\log\left(\left(\frac{T_d}{T_{ncp}}\right)\left(\frac{T_a}{T_{1.3}}\right)\left(\frac{t}{t_f}\right)^{1/2}\left(\frac{s_{d,z}}{s}\right)^{1/2}\left(\frac{\text{fwhm}_f}{\text{fwhm}}\right)\right) \tag{3}$$

We will assume that the mission of DECam in the network of telescopes following up LIGO is to cover a wide area deep. We take a fixed exposure time of 30 seconds, and find that for i-band m = 22.3, $10\sigma$for a point source (For t=90 seconds, m=23.0).

$$m_i = 22.3 + \Delta m \tag{4}$$

$$= 22.3 + 2.5\log\left(\left(\frac{T_d}{T_{ncp}}\right)\left(\frac{T_a}{T_{1.3}}\right)\left(\frac{t}{30}\right)^{1/2}\left(\frac{0.9''}{\text{fwhm}}\right)\right) + \frac{1}{2}(m_{sky} - 22.0) \tag{5}$$

The only term we hold fixed will be t, at t=30 seconds. Then we must build maps at a given time for m_i.

$$m_i = 22.3 + 2.5 \log \left(\left(\frac{T_d}{T_{ncp}} \right) \left(\frac{T_a}{T_{1.3}} \right) \left(\frac{0.9''}{\mathrm{fwhm}} \right) \right) + \frac{1}{2}(m_{sky} - 22.0) \quad (6)$$

Elements of the limiting magnitude map

Hard limits There are two sets of time dependant hard limits. The first is the horizon at zenith distance = 90°. Second is the fact that many telescopes have limits in zenith distance less than 90°that vary with hour angle. For the moment we ignore both of these; other effects will model the opacity of the earth, and the telescopes limits have to asked for.

Transmission due to galactic dust The Planck map comes as a healpix map, with the values of the pixels being $\tau_{350\mu m}$ multiplied by a normalization to take it to E(B-V), the "reddening". This is best understood as A_B-A_V, where the A is the "extinction" in a given filter.

Table 1: values to multiply E(B-V) to get A_f

DES filter	A_f/E(B-V)
u	4.544
g	3.764
r	2.765
i	2.483
z	1.935
y	1.515

The transmission dues to dust is $T_d = 10^{-0.4 \frac{A_i}{E(B-V)} E(B-v)_{planck}}$ where A_i is extinction in the i-band, so the transmission relative to the fiducial is

$$\frac{T_d}{T_{min}} = 10^{-0.4 \frac{A_i}{E(BV-V)} (E(B-V) - 0.0041)} \quad (7)$$

The dust map comes from the pages at `http://wiki.cosmos.esa.int/planckpla/index.php/CMB_and_astrophysical_component_maps#Thermal_dust_emission`, and more specifically at `http://pla.esac.esa.int/pla/aio/product-action?MAP.MAP_ID=HFI_CompMap_ThermalDustModel_2048_R1.20.fits`, the column of interest is EBV.

We downgrade the resolution to 512 (downgrading takes the mean of the input pixels to calculate the output pixel), and converted it to equatorial coordinates.

Transmission due to the atmosphere In a very simple model the transmission due to the atmosphere can be described as

Table 2: rough atmospheric extinction terms

DES filter	k_f
u	0.58
g	0.18
r	0.09
i	0.08
z	0.08
y	0.08

$$\frac{T_a}{T_{a,f}} = 10^{-0.4 k_i X} / 10^{-0.4 \cdot 1.3 k_i} = 10^{-0.4 k_i (X - 1.3)} \quad (8)$$

where k_i is the (first order) atmospheric extinction in the i-band, and X is the airmass. The fiducial airmass is 1.3.

Since we're workind down to zd = 90°, we shouldn't use the X=sec(zd) approximation to airmass. Instead we use one due to Young (Young, 1994) which is quoted to have an maximum error of 0.004 down to zd=90°:

$$X = \frac{1.002432 \cos^2(zd) + 0.148386 \cos(zd) + 0.0096467}{\cos^3(zd) + 0.149864 \cos^2(zd) + 0.0102963 \cos(zd) + 0.000303978} \quad (9)$$

THE PSF FWHM The full width half max of the PSF is, statistically, a filter dependant powerlaw in zenith distance.

$$\frac{r}{r_f} = \left(\frac{\lambda}{\lambda_f}\right)^{-0.2} \left(\frac{X}{1.3}\right)^{3/5} \tag{10}$$

THE SKY SURFACE BRIGHTNESS The dark time sky surface brightness m is a filter dependant power law. We have a simple dark sky model at present:

$$m_{sky} = m_{sky,f}(1.3) - c_f\,(zd - 40^\circ) \tag{11}$$

where $m_{sky,f}$ and c_f are empiracal measurements of the sky brightness as a function of zenith distance at CTIO.

This is usally swamped by the contribution of moon light or sunlight, so the actual computation involves a atmospheric scattering model. We have such a model, but have not implemented it in this context.

IN SUMMARY, the limiting magnitude maps are in good shape, needing only the moon model to be incorporated.

Table 3: rough atmospheric extinction terms

filter	$m_{sky}(1.3)$	c_f *(mags/degree)*
u		
g	22.0	0.0114
r	20.9	0.0121
i	20.0	0.0081
z	18.6	0.0081
y	18.6	0.0150

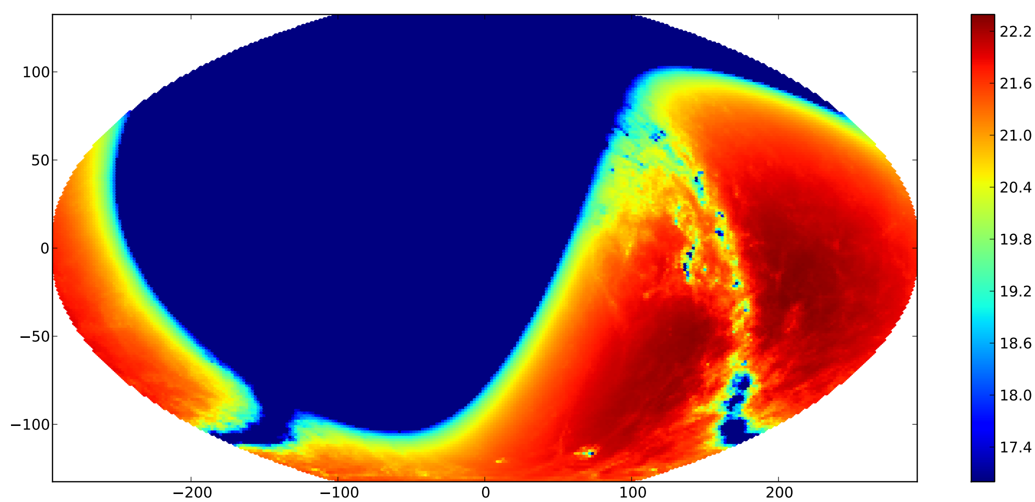

Figure 1: The limiting magnitude map for event id 1087, MJD = 55435.60552.

Probability of detection

The LIGO maps present P_s, the probability the source is in the pixel. We wish to calculate

$$P_f = P_s * P_d \tag{12}$$

where P_d is the probability of object detection in that pixel, and P_f is the probability of finding the object.

P_d is heavily time dependant as the earth rotates, revolves around the sun, and the moon revolves around the earth, thought these are solved problems. P_d is model dependant as one must have a model for the counterpart that one trying to detect.

Let's start with this form.

$$P_d = \int_{m(10\sigma)}^{15} P(m|s,t)P(d|m,t)P(r|m)dm \tag{13}$$

The $P(m|s,t)$ is the probability of magnitude m given the source model s and the time, t; m=15 is taken as a saturation limit. The $P(d|m,t)$ is the probability of detection given magnitude m and the time, t. The $P(r|m)$ is the probability of recognizing the detection given the detection magnitude; this is related to a false positive rate.

Recognizing the detection given the magnitude We will model $P(r|m)$ as a false positive rate problem related to stellar density. If the targeted region is too full of stars there is likely to be far too many false positives to be able to understand them, let alone follow them up.

The star density on the sky is known. We can use the 2MASS J-band star counts in units of stars/sq-degree, normalized simply by linear range of 0-1 between min and max. In detail the data comes from the Nomad catalog and we cut at J < 16.0, and fill each healpix pixel with the star counts. Surface density is then found by dividing by the (contant) pixel area.

The probability is taken to be $P(r|m) = 0.0$ at 610 stars/sq-degree (roughly that of the galactic anticenter) and $P(r|m) = 1.0$ at 10 stars/sq-degree (roughly that of the south galactic pole), linear in surface density. This gives a probability map that resembles that map of nearby galaxies from 2MPZ.

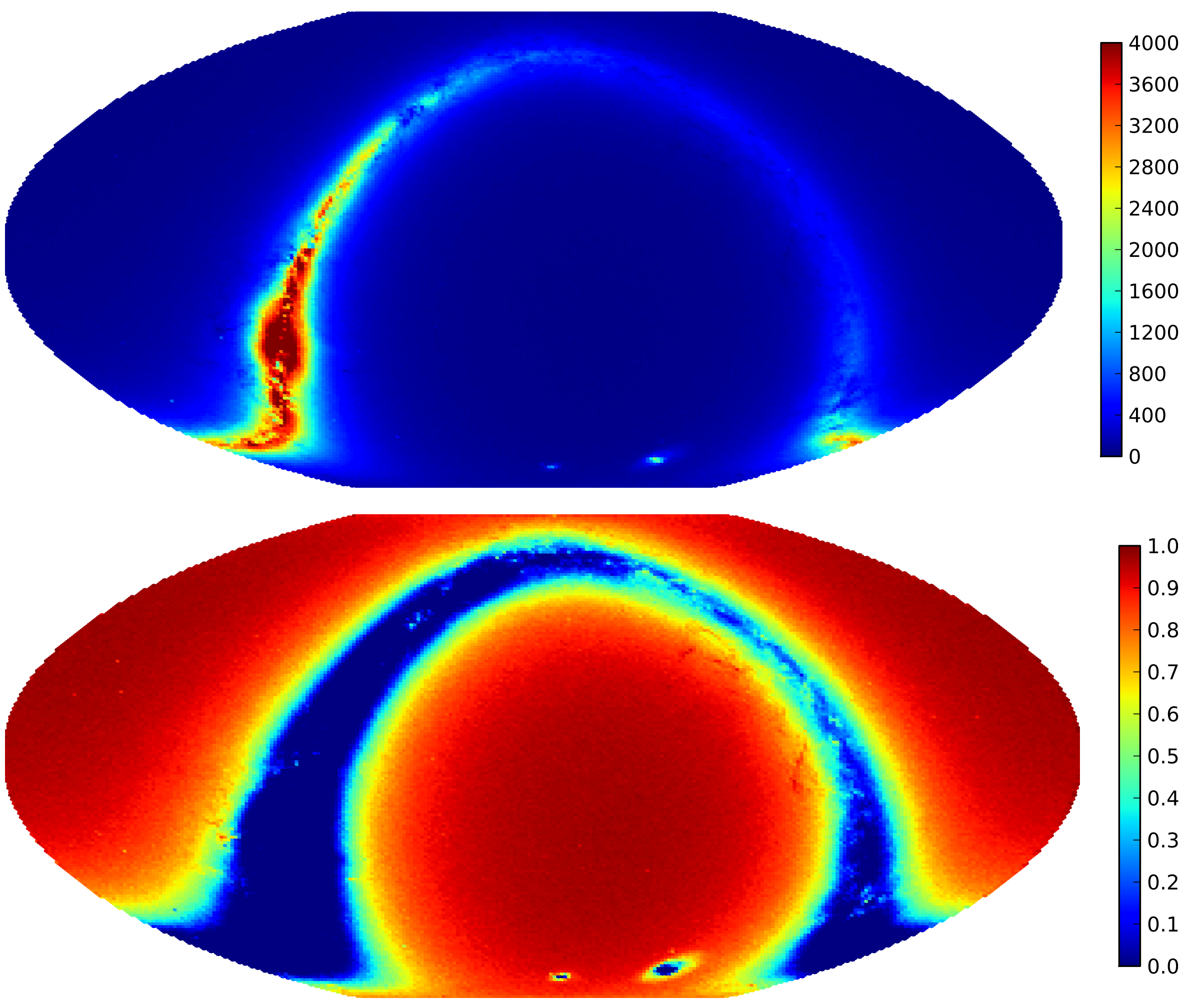

Figure 2: Top: star density map, using 2MASS J < 16.0 star counts, in star/sq-degree. Bottom: $P(r)$ map, assumed independent of magnitude, with a scaling from 0 to 1 set from galactic anticenter to south galactic cap.

The probability of the magnitude given the source model

We have to assume a specific source model for $P(m|s,t)$. The better specified the model, the easier it is to filter out unwanted objects in our search field, and the easier it is to miss counterparts of a type not well described by the model. We'll use the kilonova model. The primary signatures of the kilonova model are the week-long decay time scale of the optical magnitudes, and the red i-z color.

The radial probability

We take two priors on the distance:

$$P_d = P_{sd} P_V \tag{14}$$

where the probability of the counterpart being at a distance d is P_d and is composed of P_{sd}, the LIGO derived source distance pdf, and P_V, the volume weighting, the astronomy derived pdf of the stellar mass.

At present, the P_d is computed from 1 Mpc to 100 Mpc in steps of 1 Mpc.

The volume weighting, P_V, can be made simplistic (uniform weighting from 1 to 100 Mpc), statistically impeccable (volume shell weightings, $p \propto d^2/d^3_{max}$), to using as much domain knowledge as possible, which involves galaxy catalogs.

At present the volume shell weighting is implemented. Research into the galaxy catalogs is next on the list.

The source distance will, I assume, be given as a pdf on distance from the LIGO data. At present this is implemented as a gaussian pdf.

The spatial probability

Making a detection given the magnitude We will assume the simpliest form for $P(det|m,t)$:

$$P(det|m,t) = \begin{cases} 1 & \text{if} \quad m >= m_{10\sigma} \\ 0 & \text{if} \quad m < m_{10\sigma} \end{cases}$$

On the other hand, our ability to reach $m_{10\sigma}$ depends on all things astronomy and is where we need time dependant maps.

Probability of a given magnitude We start with the Barnes and Kasen models (Barnes and Kasen, 2013) and simply find the mean and standard deviation of the i-band magnitudes of the models in table 1: $M_i = -14.1 \pm 1.3$.

The probability is then

$$P = \int_{M_{min}}^{M_{max}} dM \int_{1}^{100} dd \quad m(d, M) P(det|m) \qquad (15)$$

where this is to be taken as for each pixel in the map, m is the apparent magnitude $m = M_i + 5\log(d_{mpc}) + 25$, and d is the distance in Mpc.

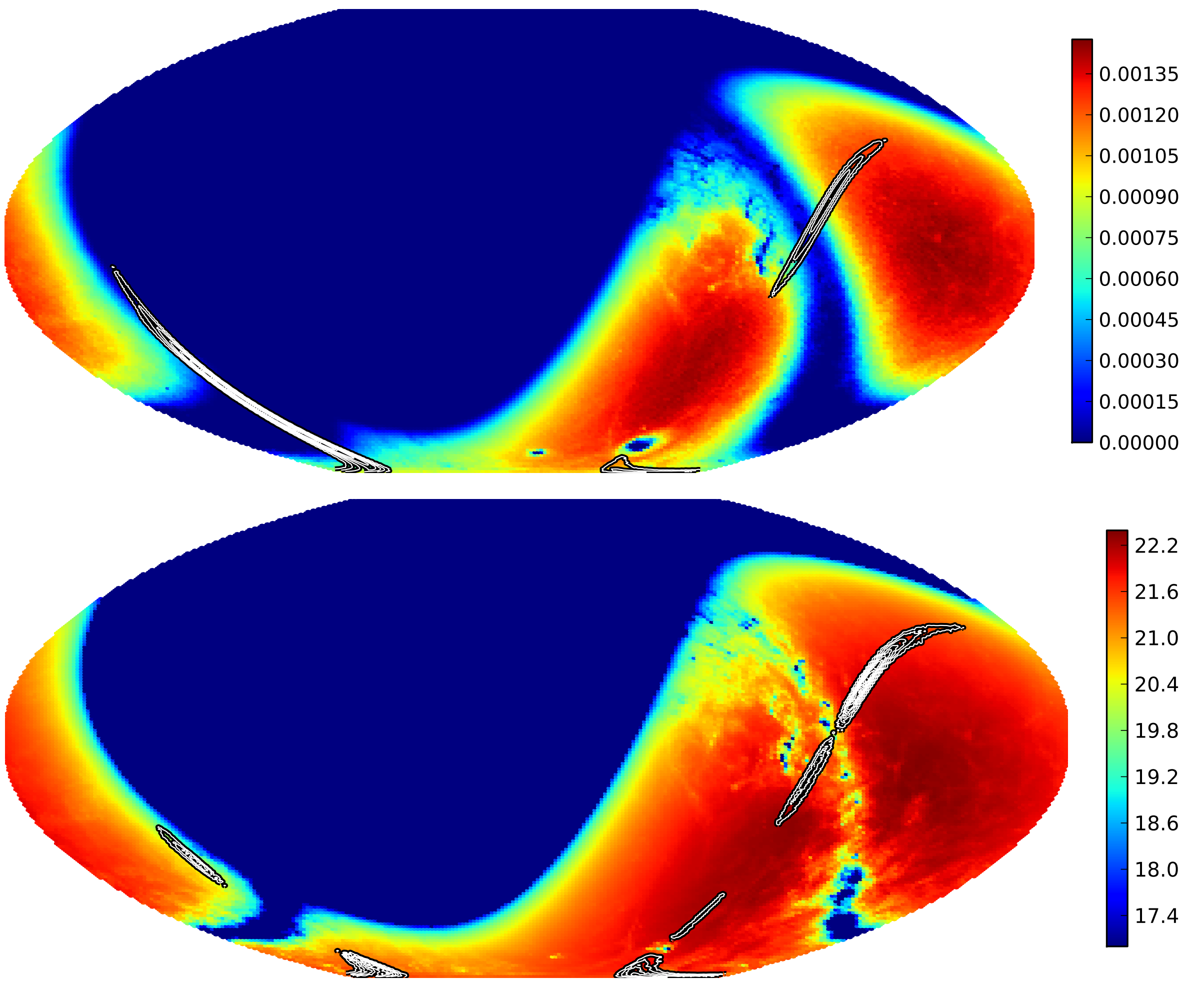

Figure 3: Top: A full probability map taking the LIGO source predicted distance as 70 ±10 Mpc, with LIGO probabilities plotted as contours. Bottom: A limiting magnitude map, with contours of counterpart finding probability $P_f = P_{LIGO} * P_d$.

The next iteration should replace the volume weighting with a galaxy weighting that is the cumulative PDF at z < 0.03 of the galaxies in the Two Micron All Sky Survey Photometric Redshift Catalog (Bilicki et al., 2014); the PDF is from the photo-z (σ_{pz} = 0.015) but if there is a spectroscopic z replace. Furthermore, if there is a real distance available from the Extragalactic Distance Database (Tully et al., 2013) for a galaxy in that pixel, place the entire pixel weight at that distance.

From probability maps to camera pointings

There is a list of DES hexes that covers the whole sky. It is easy to imagine summing the probability inside each hex for a given time dependant map and ranking the hexes.

Exactly how one connects the time dependant maps is a TBD; most likely by considering each night separately inside a survey strategy model. When one thinks about connecting the maps inside a given night, one has to consider the overhead times for moving the telescope. All this is TBD.

References

Barnes, J. and Kasen, D. (2013). Effect of a High Opacity on the Light Curves of Radioactively Powered Transients from Compact Object Mergers. *Astrophysical Journal*, 775:18.

Bilicki, M., Jarrett, T. H., Peacock, J. A., Cluver, M. E., and Steward, L. (2014). Two Micron All Sky Survey Photometric Redshift Catalog: A Comprehensive Three-dimensional Census of the Whole Sky. *Astrophysical Journal, Supplement*, 210:9.

Singer, L. P., Price, L. R., Farr, B., Urban, A. L., Pankow, C., Vitale, S., Veitch, J., Farr, W. M., Hanna, C., Cannon, K., Downes, T., Graff, P., Haster, C.-J., Mandel, I., Sidery, T., and Vecchio, A. (2014). The First Two Years of Electromagnetic Follow-up with Advanced LIGO and Virgo. *Astrophysical Journal*, 795:105.

Tully, R. B., Courtois, H. M., Dolphin, A. E., Fisher, J. R., Héraudeau, P., Jacobs, B. A., Karachentsev, I. D., Makarov, D., Makarova, L., Mitronova, S., Rizzi, L., Shaya, E. J., Sorce, J. G., and Wu, P.-F. (2013). Cosmicflows-2: The Data. *Astronomical Journal*, 146:86.

Young, A. T. (1994). Air mass and refraction. *Applied Optics*, 33:1108–1110.